GRAVITY AND MAGNETIC FORWARD MODELLING OF THE REGOLITH

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INTRODUCTION

The regolith in Australia can be described as everything from fresh rock to fresh air (Eggleton 2001). Recently, there have been many advances in gravity and magnetic acquisition methods (Hammer 1983, Doll *et al.* 2001, Dransfield *et al.* 2001, Hammond & Murphy 2003), and it may even be possible to use gravity and magnetic methods to explore within the regolith (Heath *et al.* 2004). With new techniques in measurement of the physical properties of the Earth comes the need for an understanding of how these processes occur in a natural environment.

In order to determine geophysical responses of regolith scenarios (and geology in general), it is necessary to create a simulation of the scenario on a computer. This paper describes a forward modelling process to calculate components of the Earth's gravity and/or magnetic field for a general regolith scenario. The process can be applied to geological scenarios in general.

To start, the formulae required to calculate the fields for a rectangular prism are listed. These can be added cumulatively in order to model complex regolith scenarios. The forward modelling methodology is then outlined, discussing the various steps of the process. The results section shows the gravity and magnetic responses of some simple objects, in order to show the validity of the formulae, and finally the gravity and magnetic response of a full regolith model is presented.

FORMULAE

Equations used to calculate the various components of the gravity or magnetic field of a rectangular prism are summarised below. Formulae (along with a discussion on the notation) are taken from various sources, mainly Heath *et al.* (2004). The reference frame is the same as in Zhang *et a.,* (2000) and Heath *et al.* (2004).

Equation (1) is used to calculate the total gravitational response of a rectangular prism. It is composed of the 3 vector components shown in equations (2) to (4). In these equations *G* is the universal gravitational constant and *m* is the mass of the prism. The variables *X*, *Y*, *Z* and *R* are all discussed in Heath *et al.* (2004).

$$G_{tgi} = \sqrt{G_x^2 + G_y^2 + G_z^2} \tag{1}$$

$$G_x = Gm\left(Z - X \tan^{-1}\left(\frac{Z}{X}\right) + X \tan^{-1}\left(\frac{YZ}{XR}\right) - Z \ln(Y+R) - Y \ln(Z+R) \right)^* \tag{2}$$

$$G_y = Gm\left(Z - Y \tan^{-1}\left(\frac{Z}{Y}\right) + Y \tan^{-1}\left(\frac{XZ}{YR}\right) - Z \ln(X+R) - X \ln(Z+R) \right)^* \tag{3}$$

$$G_z = Gm\left(X - Z \tan^{-1}\left(\frac{X}{Z}\right) + Z \tan^{-1}\left(\frac{XY}{ZR}\right) - X \ln(Y+R) - Y \ln(X+R) \right)^* \tag{4}$$

Recently there has been much work in the acquisition of the tensor components of the Earth's gravitational field (Dransfield *et al.* 2001, Hammond & Murphy 2003). The tensor component responses of a prism are the various differentials of the vector components of the field (Blakely 1995). There are five independent components of the field, shown in equations (5) to (9).

$$G_{xx} = Gm\left(\tan^{-1}\left(\frac{YZ}{XR}\right) \right)^* \tag{5}$$

$$Gyy = Gm\left(\tan^{-1}\left(\frac{XZ}{YR}\right) \right)^* \tag{6}$$

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$$G_{xy} = Gm(\ln(Z+R))^* \quad (7)$$

$$Gxz = Gm(\ln(Y+R))^* \quad (8)$$

$$Gyz = Gm(\ln(X+R))^* \quad (9)$$

The corresponding magnetic responses can be derived by the application of Poisson's Relationship (Blakely 1995). Poisson's Relationship states that the magnitude of a magnetic field at a point is proportional to the differential of the gravitational field at that point. The direction of the differential is the direction of the inducing magnetic field, and the proportionality constants are the magnetic susceptibility of the object, the strength of the inducing magnetic field, the mass of the object, the universal gravitational constant, and the magnetic permeability.

By replacing the single differential with three weighted partial derivatives, it is possible to derive analytically the equations for the magnetic response of a rectangular prism directly from the gravitational equations. In the following equations F_e is the strength of the inducing magnetic field, k is the magnetic susceptibility of the object and α, β and γ represent the three weights that alter depending on the direction of the field. The total magnetic intensity is shown in equation (10), and the three vector components in equations (11) to (13).

$$B_{tmi} = \sqrt{B_x^2 + B_y^2 + B_z^2} \quad (10)$$

$$B_x = \frac{F_e k}{Gm}\left(\alpha\left(\tan^{-1}\left(\frac{YZ}{XR}\right)\right) + \beta(\ln(Z+R)) + \gamma(\ln(Y+R))\right)^* \quad (11)$$

$$B_y = \frac{F_e k}{Gm}\left(\alpha(\ln(Z+R)) + \beta\left(\tan^{-1}\left(\frac{XZ}{YR}\right)\right) + \gamma(\ln(X+R))\right)^* \quad (12)$$

$$B_z = \frac{F_e k}{Gm}\left(\alpha(\ln(Y+R)) + \beta(\ln(X+R)) + \gamma\left(\tan^{-1}\left(\frac{XY}{ZR}\right)\right)\right)^* \quad (13)$$

The three weights alpha, beta and gamma can be determined by the trigonometric equations (14) to (16), where I is the inclination of the inducing field and D is the declination.

$$\alpha = \frac{\cos D}{\tan I + \cos D - \sin D} \quad (14)$$

$$\beta = A\tan D \quad (15)$$

$$\gamma = 1 - \alpha - \beta \quad (16)$$

As with gravity, it is now possible to acquire magnetic tensor data (Doll *et al.* 2001, Gamey *et al.* 2002). This data may be useful for regolith exploration (Heath *et al.* 2004), and so the equations governing the five independent tensor responses of a rectangular prism are shown in equations (17) to (21).

$$B_{xx} = \frac{F_e k}{Gm}\left(\alpha\left(\frac{-\frac{YZ}{R}\left(\frac{1}{X^2}+\frac{1}{R^2}\right)}{1+\left(\frac{YZ}{XR}\right)^2}\right) + \beta\left(\frac{\frac{Z}{XR}\left(1-\frac{Y^2}{R^2}\right)}{1+\left(\frac{YZ}{XR}\right)^2}\right) + \gamma\left(\frac{\frac{Y}{XR}\left(1-\frac{Z^2}{R^2}\right)}{1+\left(\frac{YZ}{XR}\right)^2}\right)\right)^* \quad (17)$$

$$B_{yy} = \frac{F_e k}{Gm}\left(\alpha\left(\frac{\frac{Z}{YR}\left(1-\frac{X^2}{R^2}\right)}{1+\left(\frac{XZ}{YR}\right)^2}\right) + \beta\left(\frac{-\frac{XZ}{R}\left(\frac{1}{Y^2}+\frac{1}{R^2}\right)}{1+\left(\frac{XZ}{YR}\right)^2}\right) + \gamma\left(\frac{\frac{X}{YR}\left(1-\frac{Z^2}{R^2}\right)}{1+\left(\frac{XZ}{YR}\right)^2}\right)\right)^* \quad (18)$$

$$B_{xy} = \frac{F_e k}{Gm}\left(\alpha\left(\frac{1}{R}\left(\frac{X}{Z+R}\right)\right)+\beta\left(\frac{1}{R}\left(\frac{Y}{Z+R}\right)\right)+\gamma\left(\frac{1}{R}\right)\right)^* \quad (19)$$

$$B_{xz} = \frac{F_e k}{Gm}\left(\alpha\left(\frac{1}{R}\left(\frac{X}{Y+R}\right)\right)+\beta\left(\frac{1}{R}\right)+\gamma\left(\frac{1}{R}\left(\frac{Z}{Y+R}\right)\right)\right)^* \quad (20)$$

$$B_{yz} = \frac{F_e k}{Gm}\left(\alpha\left(\frac{1}{R}\right)+\beta\left(\frac{1}{R}\left(\frac{Y}{X+R}\right)\right)+\gamma\left(\frac{1}{R}\left(\frac{Z}{X+R}\right)\right)\right)^* \quad (21)$$

Finally, two tensor responses (one gravitational and one magnetic) are especially important for exploration, as they generally show easily interpretable geologic features (Lane 2004). These are the derivatives in the z direction of the vertical vector components of the field. They can be calculated from the independent components of the tensor, given by equations (22) and (23).

$$G_{zz} = -G_{xx} - G_{yy} \quad (22)$$

$$B_{zz} = -B_{xx} - B_{yy} \quad (23)$$

METHOD

The geological volume to be simulated is divided up into equally sized rectangular prisms. These rectangular prisms, known as voxels, are the basic units of the model. Each voxel is designated a value for its mass and (if calculating the magnetic response) its magnetic susceptibility. This in effect creates a three-dimensional matrix, otherwise known as a matrix array.

All constants need to be set in the computer. These are primarily the universal gravitational constant and the height of the measurement plane above the surface. If the magnetic responses are desired, additional constants need to be part of the input. These are the inducing magnetic field strength and the inclination and declination of the field. The inclination and declination of the field are used to determine the three weights in equations (14) to (16). Once all constants are set, processing can begin.

At this point, the user should select which particular gravity or magnetic component they would like to observe the response of. This is simply a case of inputting the correct equation(s) into the algorithm. G_z (4) and B_{tmi} (10) are the two most commonly acquired fields (Lane 2004, Telford *et al.* 1995).

A position of measurement is selected, and from there, the cumulative gravity or magnetic responses of all the voxels in the model is calculated at that point. This requires a loop in the x, y and z direction to cover the three dimensions of the model. Once this has been determined, the value is stored in the computer memory for that position, and a new measurement position is chosen. This process can easily be automated, by simulating a grid as the measurement plane in the x and y directions. For regolith scenarios a measurement spacing of one metre is quite effective (Heath *et al.* 2004).

Once the gravity and/or magnetic field components have been measured at each point, the loops are terminated, and the data is plotted as a surface. At this point all data should be saved, including the workspace, for use in further processing.

A flow chart outlining this entire procedure is shown in Figure 1.

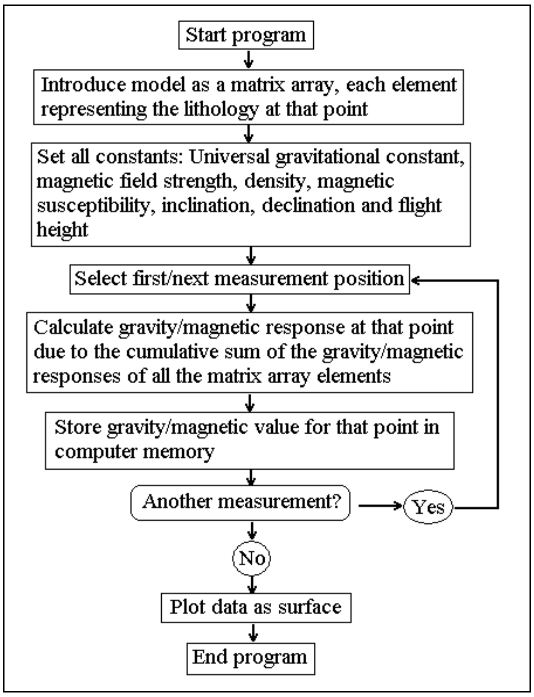

Figure 1: Flowchart showing the forward modelling process.

Examples of the processing shown here include some singular rectangular prisms to show that the algorithm works, and two different responses of the same regolith model. The regolith model is based on the simulation introduced in Heath *et al.* (2004).

RESULTS

Figure 2 shows the gravitational response (G_z) of a rectangular prism buried at 60 metres, with dimensions 1000 by 1000 metres. This simulation compares well to examples given in Telford *et al.* (1995). Figure 3 shows the magnetic response (B_{tmi}) of a rectangular prism, buried at 8 metres, with dimensions 8 by 8 metres, which again compares well to examples given in Telford *et al.* (1995). Figure 4 shows a magnetic tensor response (B_{zz}) of the regolith model introduced in Heath *et al.* (2004). The boundary between the different regolith units is clearly visible.

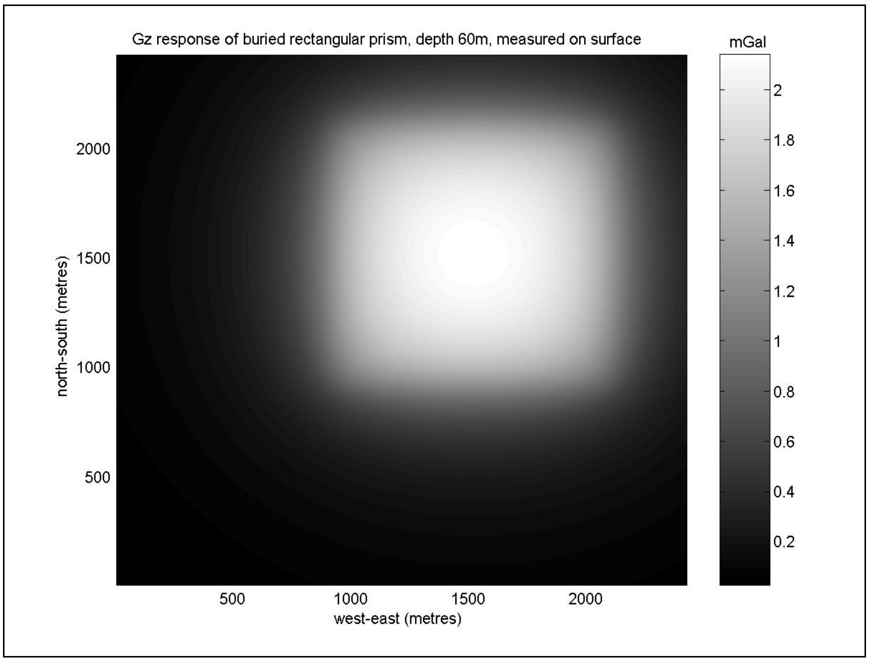

Figure 2: The Gz response due to a buried prism. The prism is buried at 60 metres, and has dimensions 1000 by 1000 metres.

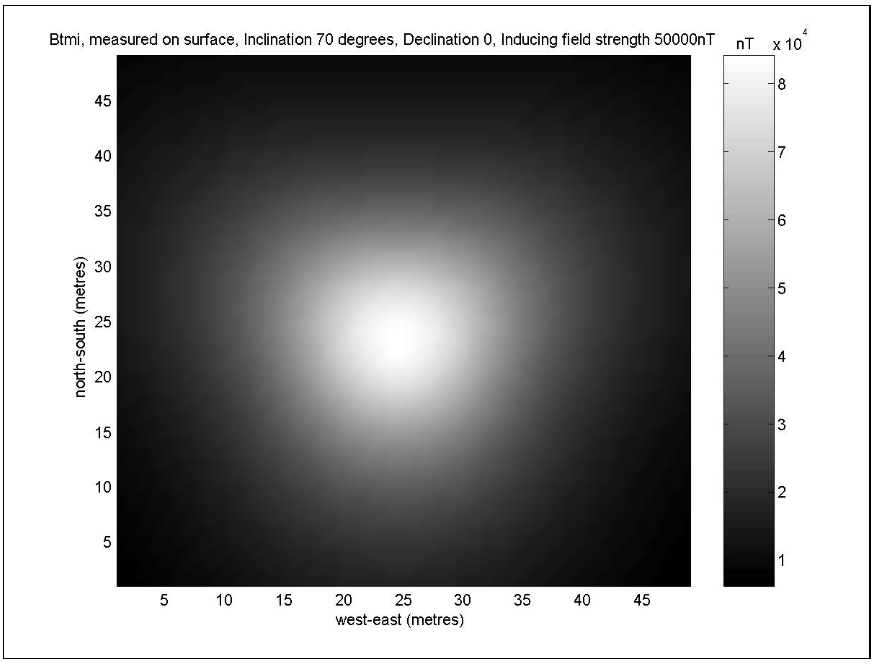

Figure 3: The Btmi response due to a buried prism. The prism is buried at 8 metres, and has dimensions 8 by 8 metres.

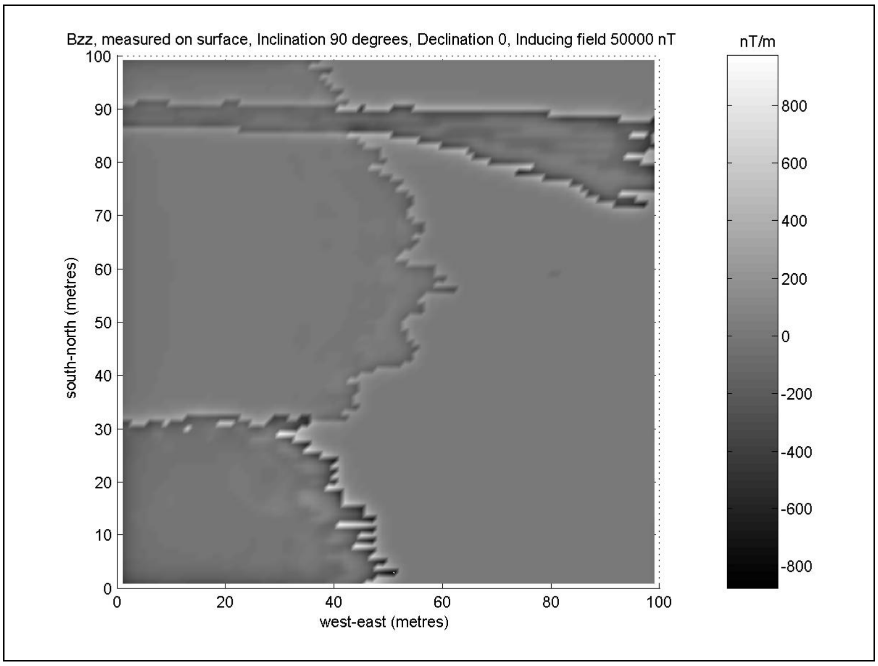

Figure 4: The Bzz response of a simulated regolith scenario. The boundaries between different units are clearly visible.

CONCLUSIONS

A geophysical forward modelling methodology has been introduced and applied to a simple models and a simulated regolith model to calculate their expected gravity and magnetic responses. The process involves dividing the geologic model into rectangular prisms, and calculating the cumulative sum of their geophysical responses at set measurement points on a plain. The formulae have been shown to correlate well with published examples, and have been used for computing the magnetic response of a regolith simulation.

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